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Packet Radio Temporary Note #172

CONNECTIVITY ISSUES IN MOBILE PACKET RADIO

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1. INTRODUCTION

Up to the present, investigations have been limited to the study of Stationary Packet Radio networks, which are partially characterized by the following two conditions:

1. Terminals, repeaters and stations are stationary.

2. The area in which terminals originate is fixed and is covered by appropriately placing the hardware at some initial time t_o.

On these assumptions, routing algorithms and initialization algorithms have been developed. In particular, the routing algorithms use information on the network configuration to derive an optimal communication procedure. In the case of general deployment, in which the repeaters and the stations are not initially aware of the network configuration, some system capacity must be spent to acquire such information and initialize the algorithms with valid parameters.

The Mobile Packet Radio system generalizes the stationary system by allowing every element of the hardware to be independently in motion in the 3-dimensional space. Conditions 1 and 2 above must be dropped and substituted with appropriate requirements, to be described below. There is a clear distinction between the two systems, not only in the definition and analysis, but also the problems one must address. For example, in a Packet Radio Network (PRNET) one is trying to determine the location of the repeaters

to achieve optimal coverage of a certain area. A repeater may be placed in a precalculated location and it is static. In the Mobile Packet Radio Network (MPRNET) one does not place the repeaters in a particular location, but they are assigned by some uncontrollable apparatus. Also, they are assumed in motion.

In PRNET a particular and predetermined zone is covered.

In MPRNET the covered zone varies with time. Requirements as to the coverage of a particular area may, however, be imposed.

In PRNET, once the system is initialized, it will remain so; only one initialization and labeling, is necessary (except if there is a high failure rate for repeaters).

In MPRNET, the system may become inoperative at any time due to motion of the devices. Hence one must address himself to the questions of how long the system is operational, or how long one must wait before one gets coverage at a specified location. Consequently, a frequent MONITORING OPERATION is required by the station.

The dynamic character of this system accentuates the need to acquire, process and proliferate network configuration information. Specifically, the connectivity matrix will be altered within a short interval, depending on the mobility parameters of the system. Thus, the initialization algorithms developed for the stationary packet radio must be activated more and more frequently, in addition to the more frequent acquisition of data, if the system is to be operational. As a consequence, the initialization capacity becomes considerable, decreasing, in conjuction, the useful information capacity.

It can be inferred from the statements above, that the available algorithms may no longer be optimal in a dynamic environment, and hence, new or modified algorithms are sought.

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2. NUMERICAL EXAMPLE

To motivate some of the definitions, concepts and observations which will follow, a concrete example is provided in this section. The mobile network under scrutiny consists of 5 repeaters moving on linear trajectories, within a two dimensional plane at low constant velocity. Such motion is completly characterized by specifying the four parameters of Table 1, where the units of time have been taken to be hours, and the units of velocity MPH. The motion is relative to the station.

Figure 1 describes graphically the trajectories followed, and Figure 2 depicts the status of the system at three samples points. The dashed circle represents the range of transmission of the devices.

The solution of the problem is furnished by a computer program which analyzes such dynamic systems. Topics of interest are:

1. The interval of time during which repeater r_i can communicate with the station S in 1-hop;

2. The interval during which repeater r_i can communicate with the station (in any number of hops);

3. The interval of time during which the entire network is connected;

4. The degradation history of the network (i.e., when at most, at least, or exactly k repeaters are unable to communicate with the station); and

5. The enhancement history of the network (i.e., when at most, etc. k repeaters return within the range of the station).

	X LOCATION AT T=0	Y LOCATION AT T=0	SLOPE	VELOCITY, MPH
REPEATER 1	-11.0000	-11.0000	-1.000	.3000
REPEATER 2	-5.0000	-5.0000	.5000	.1000
REPEATER 3	-5.0000	5.0000	-5.0000	.2000
REPEATER 4	15.0000	0.0000	2.0000	-.2500
REPEATER 5	11.0000	11.0000	1.0000	.2000

TABLE 1

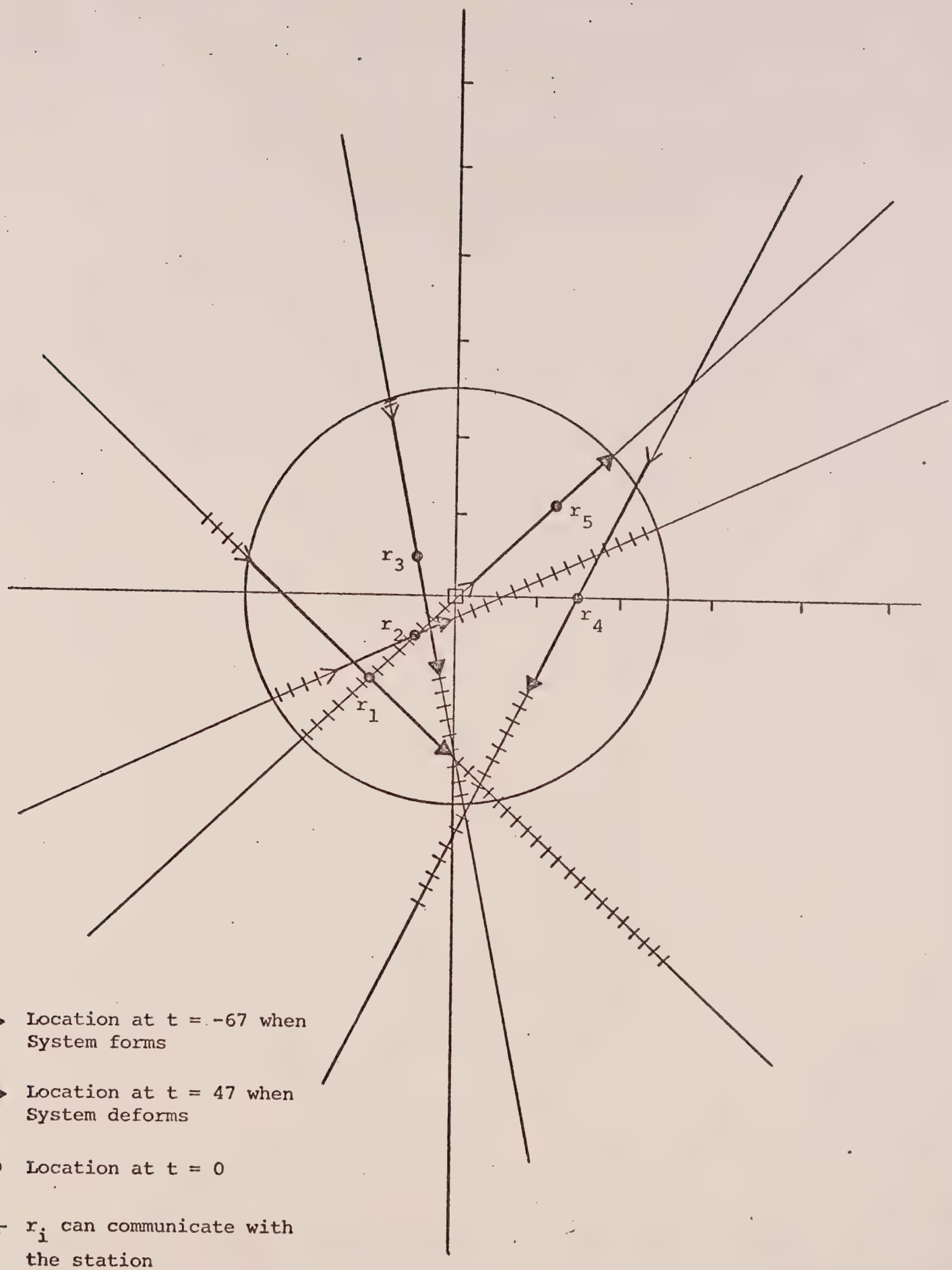

FIGURE 1: TRAJECTORIES FOLLOWED BY REPEATERS

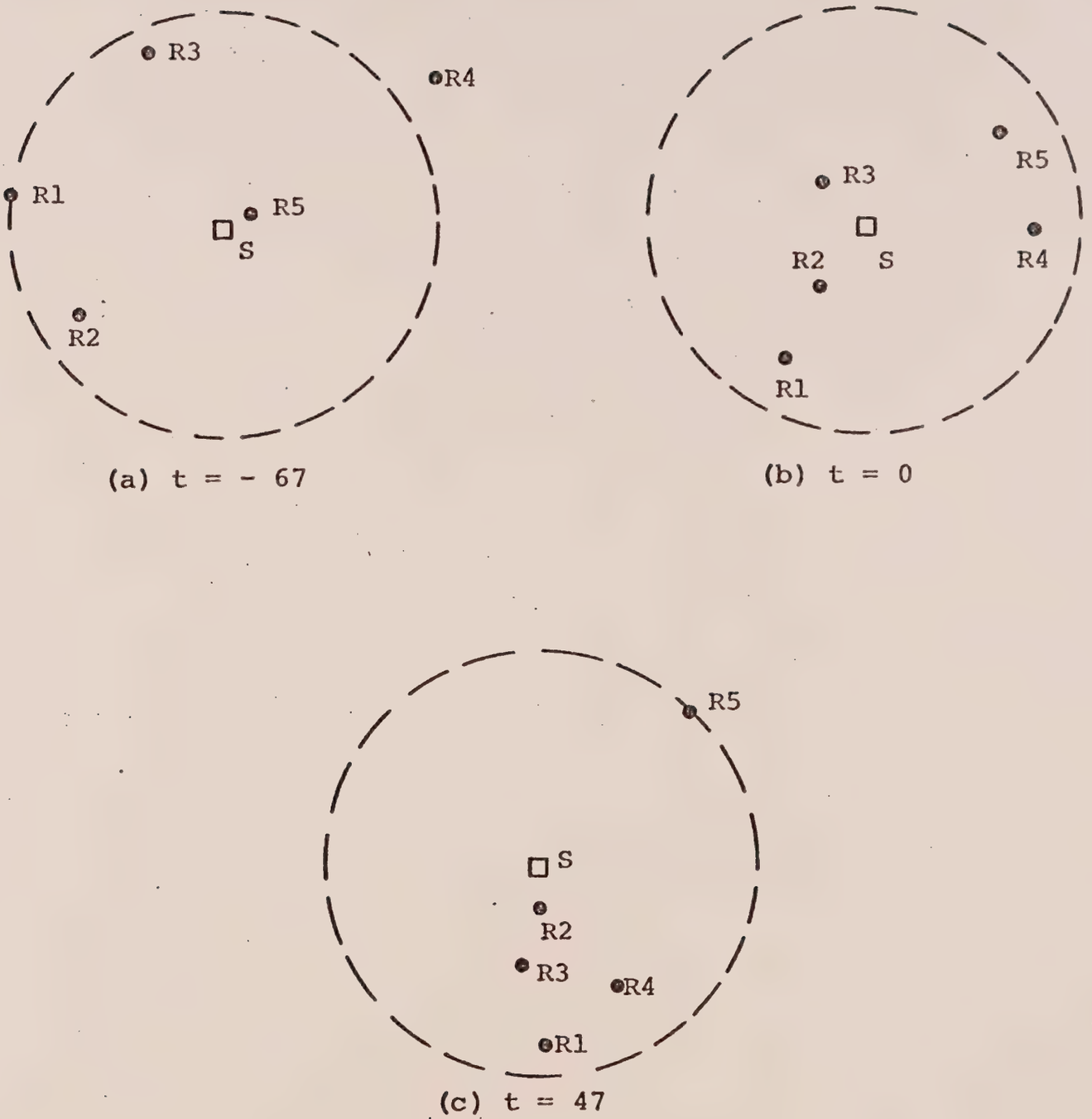

FIGURE 2: STATUS OF THE SYSTEM AT THREE SAMPLE POINTS

Table 2 shows the results for items 1, 2, 3, and 4b above for the example at hand.

The following sections will show how such information can be obtained in a general mobile network.

	(1)	(2)	(3)	(4b)
r_1	(-65.23, 65.23)	(-93.14, 151.95)		1:(-∞, -67.21) ∪ (47.21,∞)
r_2	(-181.91, 316.08)	(-202.78, 316.08)		2:(-∞, 93.14) ∪ (151.95,∞)
r_3	(-94.03, 152.86)	(-94.03, 152.86)	(-67.21, 47.12)	3:(-∞, 94.03) ∪ (152.86,∞)
r_4	(-57.54, 11.21)	(-67.21, 152.86)		4:(-∞, -202.78) ∪ (152.86,∞)
r_5	(-202.78, 47.21)	(-202.78, 47.21)		5:(-∞, -202.78) ∪ (316.08,∞)

(1) Interval during which there is direct communication with S.

(2) Interval during which there is communication with S.

(3) Interval during which the network is connected.

(4) Interval during which at least k repeaters are out of range.

TABLE 2

3. MATHEMATICAL FRAMEWORK

In this section, we introduce the mathematical framework for the MPRNET.

Definition: [Initial Operating Region]

Let $O(t_o)$ be an open 3 dimensional region, which we assume to be the inside of a cube.

Definition: [Hardware]

Let $S = \{S_1, S_2, \ldots, S_n\}$, $R = \{r_1, r_2, \ldots, r_m\}$,

$\tau = \{\tau_1, \tau_2, \ldots, \tau_L\}$ be sets satisfying:

a. $S \cap R = \phi$, $S \cap \tau = \phi$, $\tau \cap R = \phi$

b. $S \subset O(t_o)$

$S \subset O(t_o)$

$\tau \subset O(t_o)$

The sets S, R, and τ correspond to the set of stations, repeaters and terminals of a packet radio system, respectively. The requirement in a,implies that each device fulfills only one function.

Definition: [Frame of Reference]

Let $X_{S_1, O(t_o)}$, $Y_{S_1, O(t_o)}$, $Z_{S_1, O(t_o)}$ be a cartesian coordinate system, with axes parallel to the sides of $O(t_o)$ and with origin at S_1.

Note that the frame of reference can be arbitrarily selected without loss of generality. However, it is convenient to select a coordinate system with the origin at station S_1 (packet radio networks under consideration are assumed to have at least one station). The set of free nodes is implied by the selection of the coordinate system, and is defined as:

Definition: [Free Nodes]

Let $\hat{S} = S - \{S_1\}$ and $F = \hat{S} \cup R \cup \tau$. The elements of F are called free nodes.

Definition: [Zone of Coverage]

Let $O(t)$ be the zone of desired coverage at time t. Possibly $O(t) = O(t_o) = O_o$ for all t.

Definition: [Area of Coverage]

Let P_{xy} be the x-y plane.

Then $P_{xy}(t) = O(t) \cap P_{xy}$ will be called the terrestrial area of desired coverage.

Definition: [Stationary Geosystem]

The system described by the definitions above is called a Stationary Geosystem, and will be denoted as:

$$\mathcal{S}_o = \{S, R, \tau, O_o, O\}$$

Definition: [Dynamic Geosystem]

Assign to each free node f ε F a velocity vector with reference to S_1 as follows:

$$\vec{v}_f^{S_1}(t) = \vec{v}_f(t), \text{ with } | \vec{v}_t(t) | = v_f(t), \text{ and } \arg(\vec{v}_f(t)) = \theta_f(t)$$

A Dynamic Geosystem is the system described by the previous definitions and the above vector assignment. It will be denoted by $\mathcal{S}_V = \{S, R, \tau, O(t), V(t)\}$ where $V(t) = \{\vec{v}_f(t) \mid f \varepsilon F\}$.

Equivalently, specification of the displacement vector of motion, in terms of the x, y, and z components, could be given.

Note that the definition requires S_1 to be fixed in the selected frame of reference, but no generality is lost by this. In fact, if in relation to some more general frame of reference, we had associated $\vec{v}'_g$ for all g ε S ∪ R ∪ τ, then in our specific frame of reference through S_1, we would have,

$\vec{v} = \vec{0}$ associated with S_1

$\vec{v}_g = \vec{v}'_g - \vec{v}'_{S_1}$ associated with g ε $\tilde{F}$

Assumption 1:

There exists a set $P_R = \{P_1, P_2, \ldots, P_z\}$ of transmission powers for repeaters and stations with a corresponding set $\mu_R = \{\mu_1, \mu_2, \ldots, \mu_z\}$ of transmission ranges (distances).

Assumption 2:

There exists a set $P_T = \{P_\nu\}$ of transmission power for terminals, with a corresponding set $\mu_T = \{\mu_\nu\}$ of transmission range. We assume that for equal levels of transmission power, equal transmission ranges are achieved. In other words, a flat terrain is considered.

The zone covered by transmission by an element $f \varepsilon F$ at level μ, is

$$B_f^{\mu}(t) = \{x \varepsilon O(t) \mid d_{f,x}(t) \leq \mu\}$$

where d is the Eucledian distance in 3 dimensions and μ is a transmission range from the set μ_R or μ_T corresponding to whether f is a repeater or a terminal.

Definition:

$M^{\mu_k}(t) = (a_{i,j}^{\mu_k}(t))$, $1 \leq i,j \leq n + m + L$ is called the connectivity matrix when

$$a_{i,j}^{\mu_k}(t) = \begin{cases} 1 \text{ if } f_i \varepsilon B_j^{\mu_k}(t) \\ 0 \text{ otherwise} \end{cases} \qquad j \leq m + n$$

$$a_{i,j}^{\mu_k}(t) = \begin{cases} 1 \text{ if } f_i \varepsilon B_j^{\nu}(t) \\ 0 \text{ otherwise} \end{cases} \qquad j > m + n$$

The top entries in the definition of $a_{i,j}$ read: "f_i is within the range of device j." A typical connectivity matrix is shown on the next page.

	S_1	S_2	S_n	r_1	r_2	...	r_m	τ_1	τ_2	...	τ_L
S_1		1		0	1		0	0	0		1
S_2		1		1	0		0	0	0		0
S_n											
r_1				1	1		1	1	0		0
r_2					1						
.											
.											
.											
r_m					1						
τ_1							1				
τ_2									1		
.											
.											
.											
τ_L											1

Observe that a partitioned matrix is obtained, but the right most lower square is completely meaningless if we do not allow direct terminal to terminal communication.

Definition: [Status]

By status of the geosystem at time t, $\sigma(t)$, we mean its connectivity matrix, the covered zone at level μ, the routing, hierarchy of repeaters, etc.

Definition: [Potentially Operational System]

The system is potentially operational at time t if the following hold:

1. "Potentially operational W.R.T. terminals:"

   O(t) is covered by balls of radius ν centered on r_i, and

2. "Potentially operational W.R.T. repeaters:"

   Consider the graph with vertices r_i and edges between r_i and r_j if $d_{r_i,r_j} \leq \mu_k$; then the graph is connected.

Let G(t) be the zone for which the system is potentially operational at t. Obviously, we would like $O(t) \subset G(t)$.

Definition: [Actually Operational System]

A system is actually operational if:

1. It is potentially operational, and

2. If the algorithms and parameters used by devices for packet transportation are such that there is a communication path between any pair of nodes. (The communication paths need not be optimal.)

This definition is motivated by the fact that even though a system may be potentially operational (the hardware is in the right position), no relabeling has yet taken place - either because the connectivity matrix in the station is not up to date or because the station has not yet completely proliferated the information to the repeaters.

Definition: [Predictable System]

A system is predictable if:

$\sigma(t) = q(\sigma(t_o), v_{f_1}(t), \ldots, v_{f_\lambda}(t))$ with q known; that is, the status at time t can be determined by the station by computation alone.

Definition: [Semi-predictable System]

A system is semi-predictable if:

$$\sigma(t) = \tilde{q}(\sigma(t_o), \vec{v}_{f_1}(t), \ldots, \vec{v}_{f_\lambda}(t))$$

with $\vec{v}_{f_i}(t) = h_i(t) + X_i(t)$ where $X_i(t)$ is a random variable with known distribution and $\tilde{q}$ is known.

Definition: [Unpredictable System]

A system is unpredictable if either q or $\vec{v}$'s are not known.

4. PROBLEMS OF INTEREST IN MPRNET

We shall initially disregard terminal communication and concentrate on the Repeater-to-Station-to-Repeater network. We also assume that for a given transmission power, the effective transmission range of repeaters is equal to that of stations.

Problem 1:

Given:

1. A Dynamic Geosystem.

2. A set of transmission powers $P_R = \{P_1,\ldots,P_z\}$ and an associated set of transmission ranges $\mu_R = \{\mu_1,\mu_2,\ldots,\mu_z\}$.

Assume that the system is potentially operational. For each $\mu_i \in \mu$ determine the time for which the system becomes non-operational of degree j; that is, j repeaters are out of range.

Problem 2:

Given:

1. As above.

2. As above.

3. A set $A = \{A_1,A_2,\ldots,A_v\}$ of routing algorithms.

For each (A_k,R_j), determine the time for which the system becomes actually non-operational, at level μ_i.

Problem 3:

Given:

1. As above.

2. As above.

3. As above.

4. A set of system capacities $C_S = (C_1,C_2,\ldots,C_v)$ and a set of initialization capacities $I_c = (I_1,I_2,\ldots,I_v)$ associated with the routing algorithms. Determine A_i for which the following is attained:

$$\max_{A}(C_i-I_i)$$

Other related problems of interest are the following.

Problem 4:

Given a terminal transmission range, a repeater transmission range, and a specified zone (area) to be covered, find the minimal number and location of repeaters such that the system is potentially operational at $t = 0$.

Problem 5:

Given the locations for r_i at t, a terminal transmission range, and a repeater transmission range, determine the covered zone (area) at $t + h$, for all $h > 0$.

Problem 6:

Given a zone (area) not covered at t, determine w such that at t + w, it will be covered.

Problem 7:

Given system mobility characteristics, compute the connectivity matrix as a function of time.

Problem 8:

Given system mobility characteristics, determine the time necessary to empirically map the connectivity matrix by packets from repeaters.

Problem 9:

Determine (and minimize) the ratio of the time for initialization, mapping of connectivity matrix, computation of connectivity matrix, relabeling, etc., to the total time.

5. CONNECTIVITY ISSUES

The remainder of this note is dedicated to the solution of Problem 1 introduced previously in the case of a predictable system. The necessary machinery and definitions are presented in the following paragraphs, after which an analytical solution is derived.

5.1 Definitions

Definition:

a. A set $\Pi \subset \P$ ($\P$= real numbers) is said to be connected if for all $x,y \in \Pi$, $(1-\alpha)x + \alpha y \in \Pi$ for all $0 \leq \alpha \leq 1$.

b. $\{\Pi_i\}_{n}^{i=1}$ are the connected components of Π if Π_i are the equivalence classes of the following equivalence relation:

$$x \sim y \text{ iff } \{(1-\alpha)x + \alpha y \mid 0 \leq \alpha \leq 1\} \subset \Pi$$

Definition:

The L spectrum of Π with connected components $\{\Pi_i\}$, $*t(\Pi)$, is the following collection:

$$*t(\Pi) = \{t_i \mid t_i = \inf_{t \in \Pi_i} t\} \qquad \text{if } \Pi \neq \phi$$

$$*t(\Pi) = \{\infty\} \qquad \text{if } \Pi = \phi$$

The points of $*t(\Pi)$ are called the death dates of Π.

Definition:

The R spectrum of a set Π with components $\{\Pi_i\}$, $t_*(\Pi)$, is the following collection:

$$t_*(\Pi) = \{t_i \mid t_i = \sup_{t\in\Pi_i} t\} \qquad \text{if } \Pi\neq\phi$$

$$t_*(\Pi) = \{-\infty\} \qquad \text{if } \Pi=\phi$$

The points of $t_*(\Pi)$ are called the resurection dates of Π. In this section, we are concerned only with systems having a single station.

Definition:

Let $R = \{r_1, r_2, \ldots, r_j\}$. The Joint Life Status (JLS), $(r_1, r_2, \ldots, r_j)$ is said to be alive at time t if every repeater can communicate with the station.

Definition:

The JLS is said to experience a k^{th} order failure, $k \leq j$, if at least k repeaters cannot communicate with the station.

Definition:

Let $r_o = S$, and $r_1, r_2, \ldots, r_j$ be the repeaters of a geosystem. The set

$$\Xi_{ik} = \{t \mid d(r_i, r_k) > \mu\},\ 0 \leq i,\ k \leq j$$

represents the time when there is no direct connection between r_i and r_k.

Figure 3 depicts the set $\equiv_{10}$ for various mobility characteristics of a 1-repeater geosystem. Figure 4 shows the sets $\equiv_{10}$, $\equiv_{20}$, $\equiv_{12}$ for a 2-repeater system.

Definition:

a. Let ${}^{i}\Pi$ be the time intervals such that repeater r_i cannot communicate with the station in any number of hops.

b. Let Π^i, Ω^i, Λ^i be the time intervals when at least, exactly, at most i repeaters, respectively, cannot communicate with the station in any number of hops.

c. Let α^i, β^i, γ^i be the time instance when exactly, at least, at most i repeaters come back into range of the station ("resurect").

In this language, the columns of Table 2 for the example of Section 2, are respectively $\equiv_{i,0}$, ${}^{i}\Pi$, $C(\Pi^1)$, Π^i.
We shall assume that the displacement vector of every repeater r_i is given; namely,

$$r_i : (x_i(t),\ y_i(t),\ z_i(t))$$

5.2 Motivational Example and General Theory

Consider a 3 repeater system. We wish to derive ${}^{1}\Pi$, the time repeater r_1 is disconnected from the station S. Such an event would materialize when there are no 1 hop connections, there are no 2 hop connections, and no 3 hop connections. Each event (except the first one) can be further subdivided into subevents, relating to the particular path structure. A brief analysis shows that:

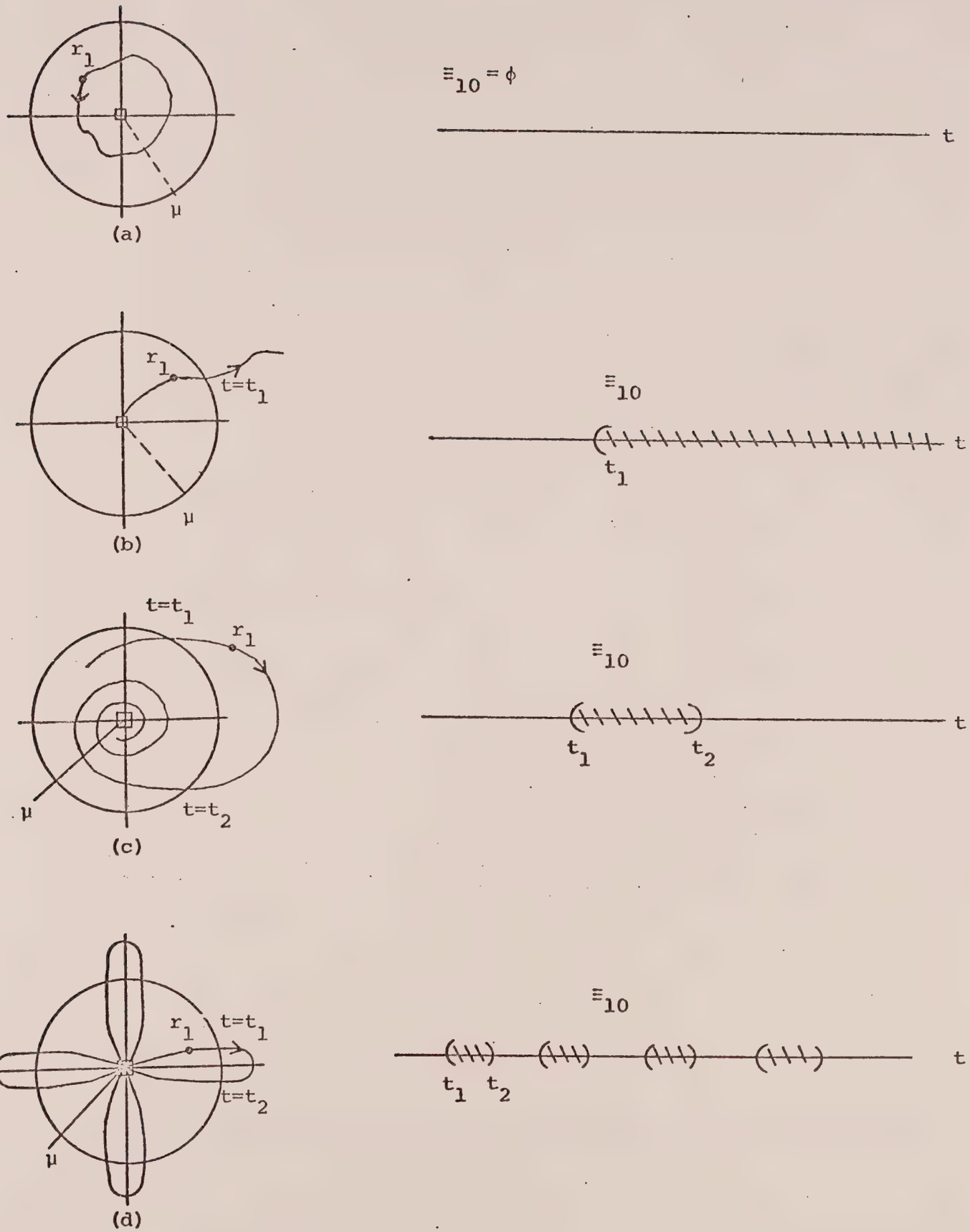

FIGURE 3: SETS Ξ_{10} FOR VARIOUS 1-REPEATER GEOSYSTEM

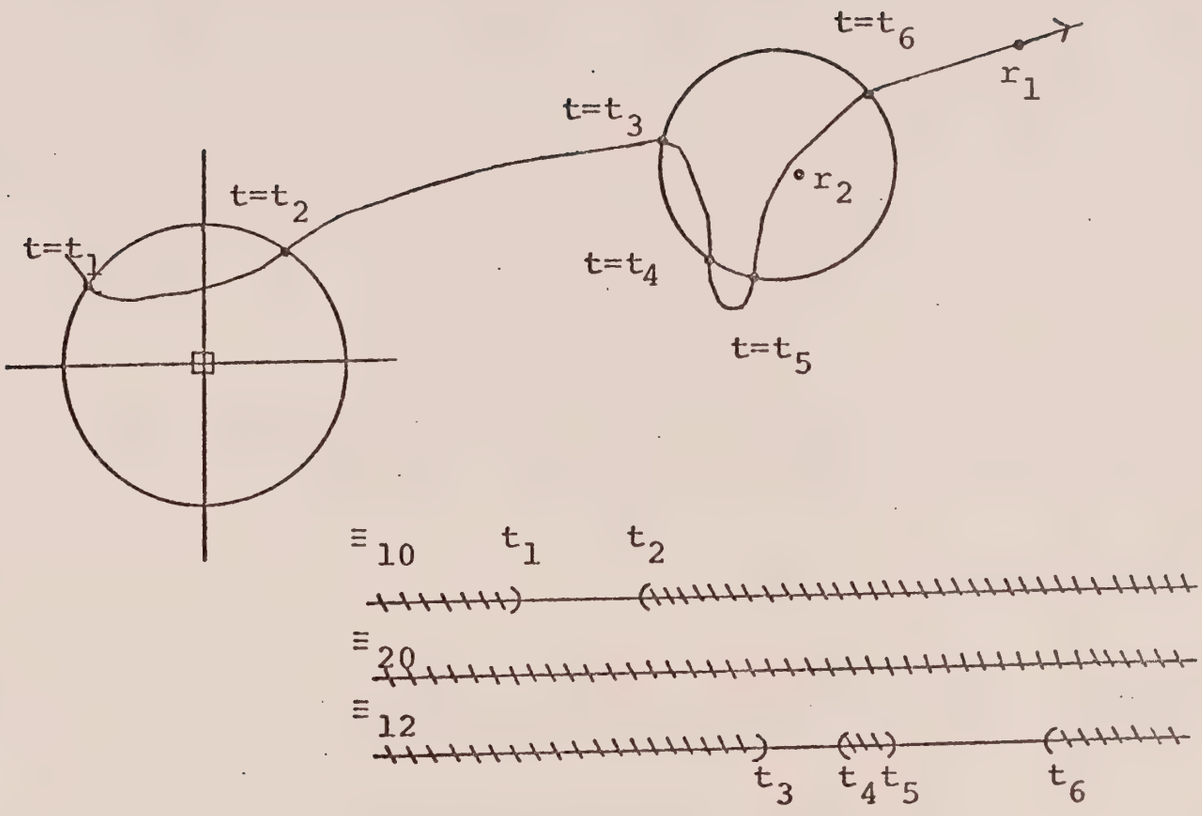

FIGURE 4: SETS $\equiv_{ij}$ FOR A 2-REPEATER GEOSYSTEM

$$^{1}\Pi = \Xi_{10} \cap (\Xi_{12} \cup \Xi_{20}) \cap (\Xi_{13} \cup \Xi_{30}) \cap (\Xi_{12} \cup \Xi_{23} \cup \Xi_{30}) \cap (\Xi_{13} \cup \Xi_{32} \cup \Xi_{20})$$

Similar expressions are obtained for $^{2}\Pi$ and $^{3}\Pi$. The JLS experiences a first order failure when either r_1 is outside the range of S, or r_2 is outside the range, or r_3 is outside the range. Thus,

$$\Pi^{1} = {}^{1}\Pi \cup {}^{2}\Pi \cup {}^{3}\Pi$$

Similarly,

$$\Pi^{2} = ({}^{1}\Pi \cap {}^{2}\Pi) \cup ({}^{1}\Pi \cap {}^{3}\Pi) \cup ({}^{2}\Pi \cap {}^{3}\Pi),$$

$$\Pi^{3} = {}^{1}\Pi \cap {}^{2}\Pi \cap {}^{3}\Pi$$

It is clear that the quantities at hand can all be derived from knowing the $\frac{1}{2}\, j(j+1)$ distinct Ξ_{ik} sets. A general methodology is now presented.

Step 1:

Derive Ξ_{ik} for all $0 \leq i \leq k \leq j$ by solving the cartesian equation:

$$\Xi_{ik} = \{t \mid (x_i(t) - x_k(t))^2 + (y_i(t) - y_k(t))^2 + (z_i(t) - z_k(t))^2 > \mu^2\}$$

Step 2:

Derive $^{h}\Pi$, $1 \leq h \leq j$ as follows:

$$h_{\Pi} = \bigcap_{k=1}^{j} \bigcap_{\substack{1 \le i_1, i_2, \ldots, i_k \le j \\ i_2 \neq i_1 \\ i_3 \neq i_2, i_1 \\ \vdots \\ i_k \neq i_{k-1}, i_{k-2}, \ldots, i_1}} (\equiv_{h, i_1} \cup \equiv_{i_1, i_2} \cup \ldots \cup \equiv_{i_k, 0})$$

Namely,

$$h_{\Pi} = \equiv_{h_o} \cap [\bigcap_{\substack{1 \le k \le j \\ k \neq h}} (\equiv_{hk} \cup \equiv_{k_o})] \cap [\bigcap_{\substack{1 \le k, L \le j \\ k \neq h \\ i \neq k, h}} (\equiv_{hk} \cup \equiv_{kL} \cup \equiv_{i_o})] \ldots$$

Step 3:

Construct Π^h, $1 \le h \le j$, as follows:

1. $\Pi^1 = \bigcup_i {}^i\Pi$.

2. $\Pi^2 = \bigcup_{1 \le i \le k \le j} ({}^i\Pi \cap {}^k\Pi)$

3. $\Pi^3 = \bigcup_{1 \le i \le k \le h \le j} ({}^i\Pi \cap {}^j\Pi \cap {}^h\Pi)$

et cetera.

Step 4:

Construct Ω^h, $1 \le h \le j-1$, as follows:

$$\Omega^h = \Pi^h - \Pi^{h+1}$$

$$\Omega^j = \Pi^j$$

Step 5:

Construct Λ^h, $1 \leq h \leq j$, as follows:

$$\Lambda^h = \bigcup_{i=1}^{h} \Omega^i$$

5.3 Properties

1. $\Pi^1 \supset \Pi^2 \supset \Pi^3 \supset \ldots \Pi^j$
For example, ${}^i\Pi \cap {}^k\Pi \subset {}^i\Pi$, so that $\Pi^2 = \cup({}^i\Pi \cap {}^k\Pi) \subset \cup {}^i\Pi = \Pi^1$

2. Π^h represents the time when h or more repeaters are outside the range of the station; Π^{h+1} represents the time when h+1 or more repeaters are outside the range of S; hence, indeed $\Omega^h = \Pi^h - \Pi^{h+1}$.

3. Ω^h's are disjoint sets.

4. Π° = time when at least zero repeaters are outside the range, so $\Pi^\circ = (-\infty, \infty)$.

5. Ω° = time exactly zero repeaters are dead; i.e., every repeater can communicate with the station; it follows that:

$$\Omega^\circ = \Pi^\circ - \Pi^1 = \P - \Pi^1 = C(\Pi^1).$$

where C(A) is the complement of set A.

6. $\Omega^\circ = \Lambda^\circ$

5.4 Complexity Considerations

We define complexity to be the number of computational steps required to carry out a given task. In this context, the term "step" does not refer to a specific operation; it could refer to a multiplication or to a union, etc.

Consider a system having j repeaters and one station. The complexity of Step 1 is $\frac{1}{2}j(j+1)$. (Step refers to the number of distinct sets to be obtained). The complexity of Step 2 is:

$$\chi = \sum_{k=1}^{j} \frac{j!}{(j-k)!} k$$

Since there are $\frac{j!}{(j-k)!}$ paths of k, each admitting k possible disconnections. The complexity of Step 3 is:

$$\sum_{k=1}^{j} \binom{j}{k} = 2^j - 1$$

(step refers to unions and intersections). Clearly, Step 2 is the most complex of the three. The following bounds are easily obtained:

$$(2j-1)j! \leq \chi \leq e.j.j!$$

Table 3 shows the numerical values of the bounds for small j. Practically speaking, exact solutions to systems having only six or less repeaters is feasible. For such a case, a CPU consumption of 200 seconds is typical.

5.5 Approximate Solutions

In view of the high computational complexity of the exact solutions, approximate solutions are sought. Two procedures are proposed.

LOWER BOUND	NUMBER OF REPEATERS	UPPER BOUND
1	n=1	2.7
6	n=2	10.8
30	n=3	48.6
168	n=4	259.2
1080	n=5	1620
7920	n=6	11664
65520	n=7	95256
604800	n=8	870912
6168960	n=9	8817984

TABLE 3

Pre-Cumulative Approximating Procedures of Degree d (Pre-CAP:d)

In this approach, a repeater r_h is declared to be dead if it is unable to communicate with the station in d hops or less. It follows that:

$$h_{\Pi} \simeq \bigcap_{k=1}^{d} \bigcap_{\substack{1 \le i_1, i_2, \ldots, i_k \le j \\ i_2 \neq i_1 \\ i_3 \neq i_2, i_1 \\ \vdots \\ L_k \neq i_{k-1}, i_{k-2}, \ldots, i_1}} (\equiv_{h,i_1} \cup \equiv_{i_1,i_2} \cup \ldots, \cup \equiv_{i_k,0})$$

Note that the outer intersection encompasses d-or-less-hop connections to the station.

Pre-Sectional Approximation Procedure of Degree d (Pre-SAP:d)

In this approach, a repeater r_h is termed dead unless there is a d-hop connection to the station. Then,

$$h_{\Pi} \simeq \bigcap_{\substack{1 \le i_1, i_2, \ldots, i_d \le j \\ i_2 \neq i_1 \\ i_3 \neq i_2, i_1 \\ \vdots \\ i_d \neq i_{d-1}, \ldots, i_1}} (\equiv_{h,i_1} \cup \equiv_{i_1,i_2} \cup \ldots, \cup \equiv_{i_d,0})$$

Under the pre-CAP:d, the computational complexity is reduced from approximately $(2j-1)j!$ to approximately j^d. The pre-CAP approach

superior to the pre-SAP approach, and under mild conditions, the pre-CAP:2 is an excellent approximation to the actual answer. With this procedure of complexity j^2, up to 30 repeater systems can be analyzed. Other approximating schemes are available.

5.6 Resurection Dates

The instance when an inactive repeater comes back into range is useful information, in particular, in a predictable system. This section addresses the issue.

It is clear that exactly one repeater resurects when the state changes from having i dead repeaters to i-1 dead. Similarly, exactly two repeaters resurect when the state changes, from having i dead repeaters to i-2 dead repeaters, etc, for i>2. Thus, it must follow that:

$$\alpha^1 = \bigcup_{i=0}^{j-1} (t_*(\Omega^{i+1}) \cap_* t(\Omega^i))$$

...

$$\alpha^p = \bigcup_{i=0}^{j-p} (t_*(\Omega^{i+p}) \cap_* t(\Omega^i))$$

Then,

$$\beta^p = \bigcup_{k=1}^{p} \alpha^k$$

$$\gamma^p = \bigcup_{k=p}^{j} \alpha^k$$

6. CONCLUDING REMARKS

The general framework of a mobile packet radio system has been given, along with the solution of basic connectivity problems. A number of related problems pertaining to initialization algorithms, reliability, delays, multistation systems, non-predictable systems, is presently under study.